EXAMINATION PAPER-IIT-JEE 2009
(QUESTION & SOLUTIONS)

PAPER – II Code-4

Part – I (CHEMISTRY)

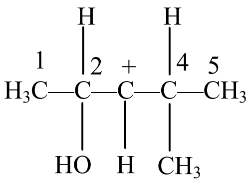
SECTION – I

Straight Objective Type

12/04/09

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. In the following carbocation, H/CH_3 that is most likely to migrate to the positively charged carbon is -



(A) CH_3 at C-4 (B) H at C-4

(C) CH_3 at C-2 (D) H at C-2

[Ans.D]

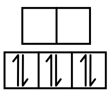
Sol. Group will migrate from C – 2 because of formation of more stable carbocation. Migrating nature of H is more than CH_3.

2. The spin only magnetic moment value (in Bohr magneton units) of $Cr(CO)_6$ is -

(A) 0 (B) 2.84

(C) 4.90 (D) 5.92

[Ans. A]

Sol. $_{26}Cr$ $3d^5$ $4s^1$ $\Rightarrow 3d^6$  t_{2g}^6 eg^0

CO is a strong field ligand.

3. The correct stability order of the following resonance structures is -

$$\underset{(I)}{H_2C=\overset{+}{N}=\overset{-}{N}} \quad \underset{(II)}{H_2\overset{+}{C}-N=\overset{-}{N}} \quad \underset{(III)}{H_2\overset{-}{C}-\overset{+}{N}\equiv N} \quad \underset{(IV)}{H_2\overset{-}{C}-N=\overset{+}{N}}$$

(A) (I) > (II) > (IV) > (III) (B) (I) > (III) > (II) > (IV)

(C) (II) > (I) > (III) > (IV) (D) (III) > (I) > (IV) > (II)

[Ans. B]

Sol. Explanation :

(1) The species with incomplete octet are less stable. Therefore II & IV are less stable than I & III.

(2) In II & IV comparison is done on the basis of E.N of atom carrying (+) charge.

(3) I is more stable than III because – ve charge is on more E.N. atoms i.e., N.

4. For a first order reaction A→P, the temperature (T) dependent rate constant(k) was found to follow the equation $\log k = -(2000)\frac{1}{T} + 6.0$. The pre-exponential factor A and the activation energy E_a,respectively, are -

(A) $1.0 \times 10^6\ s^{-1}$ and 9.2 kJ mol^{-1}

(B) 6.0 s^{-1} and 16.6 kJ mol^{-1}

(C) $1.0 \times 10^6\ s^{-1}$ and 16.6 kJ mol^{-1}

(D) $1.0 \times 10^6\ s^{-1}$ and 38.3 kJ mol^{-1}

[Ans. D]

Sol. $k = Ae^{-Ea/RT}$

or, $\ell nk = \ell nA - Ea/RT$

or, $\log k = \log A - \frac{Ea}{2.303R} \times \frac{1}{T}$

Given, $\log k = -(2000) \times \frac{1}{T} + 6$

$\log A = 6 \Rightarrow A = 10^6\ sec^{-1}$.

$\frac{E_a}{2.303R} = 2000$

$E_a = \frac{2000 \times 2.303 \times 8.314}{1000} = 38.3\ kJ\ mol^{-1}$.

SECTION – II

Multiple Correct Answers Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

5. The nitrogen oxide(s) that contain(s) N-N bond(s) is (are) -

(A) N_2O (B) N_2O_3

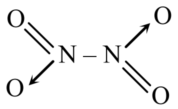
(C) N_2O_4 (D) N_2O_5

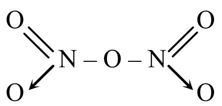
[Ans. A,B,C]

Sol. A N_2O N – N – O

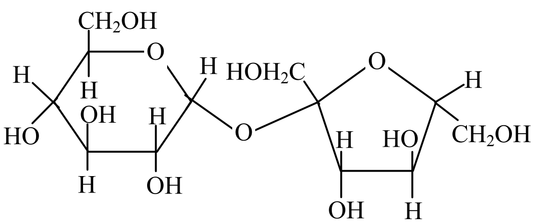
B N_2O_3 

asymmetrical form symmetrical form

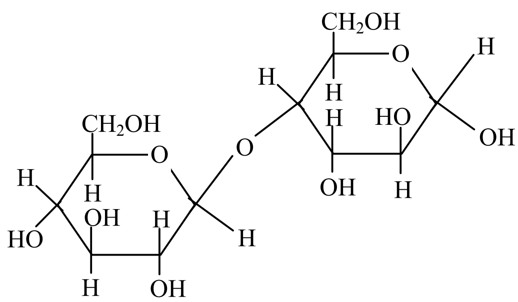
C N_2O_4 

D N_2O_5 

6. The correct statement(s) about the following sugars X and Y is (are) -



X



Y

(A) X is a reducing sugar and Y is a non-reducing sugar

(B) X is a non-raducing sugar and Y is a reducing sugar

(C) The glucosidic linkages in X and Y are α and β, respectively

(D) The glucosidic linkages in X and Y are β and α, respectively

[Ans. B,C]

Sol. In 'X' there is no free $\rangle C = O$ group and α-linkages are present while in 'Y' free $\rangle C = O$ and β-linkage is present.

7. In the reaction

$$2X + B_2H_6 \rightarrow [BH_2(X)_2]^+ [BH_4]^-$$

the amine(s) X is (are)-

(A) NH_3 (B) CH_3NH_2

(C) $(CH_3)_2NH$ (D) $(CH_3)_3N$

[Ans.A]

Sol. $B_2H_6 + NH_3 \xrightarrow[\text{low temperature}]{\text{excess } NH_3} B_2H_6 \cdot 2NH_3$

ionic compound can be represented as $[BH_2(NH_3)_2]^+[BH_4]^{-1}$

8. Among the following, the state function(s) is (are)-

(A) Internal energy (B) Irreversible expansion work

(C) Reversible expansion work (D) Molar enthalpy

[Ans.A,D]

Sol. State functions are independent of path

9. For the reduction of NO_3^- ion in an aqueous solution, E^0 is + 0.96V. Values of E^0 for some metal ions are given below -

$V^{2+}(aq) + 2e^- \rightarrow V$ $E^0 = -1.19V$

$Fe^{3+}(aq) + 3e^- \rightarrow Fe$ $E^0 = -0.04V$

$Au^{3+}(aq) + 3e^- \rightarrow Au$ $E^0 = +1.40V$

$Hg^{2+}(aq) + 2e^- \rightarrow Hg$ $E^0 = + 0.86V$

The pair(s) of metals that is (are) oxidized by NO_3^- in aqueous solution is (are)

(A) V and Hg (B) Hg and Fe

(C) Fe and Au (D) Fe and V

[Ans.A,B,D]

Sol. $V + NO_3^- \rightarrow V^{2+} + NO$ $E^0_{cell} = 0.96 - (-1.19) = 2.15$ V $\Rightarrow$ spontaneous

$Fe + NO_3^- \rightarrow Fe^{3+} + NO$ $E^0_{cell} = 0.96 - (-0.04) = 1$ V $\Rightarrow$ spontaneous

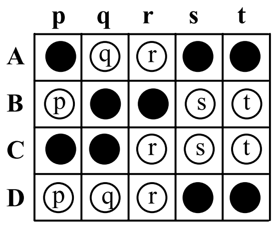
$Hg + NO_3^- \rightarrow Hg^{2+} + NO$ $E^0_{cell} = 0.96 - (0.86) = 0.1$ V $\Rightarrow$ spontaneous

SECTION – III

Matrix - Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labeled p, q, r, s and t. any given statement in **Column I** can have correct matching with **ONE OR MORE** statements (s) in **column II**. The appropriate bubbled corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following.



10. Match each of reactions given in **Column I** with the corresponding product(s) given in **ColumnII**.

Column I	**Column II**
(A) Cu + dil HNO_3	(p) NO
(B) Cu + conc HNO_3	(q) NO_2
(C) Zn + dil HNO_3	(r) N_2O
(D) Zn + conc HNO_3	(s) $Cu(NO_3)_2$
	(t) $Zn(NO_3)_2$

[Ans. (A) → p, s ; (B) → q, s ; (C) → p, t ; (D) → q, t]

Sol. $3Cu + 8HNO_3 \longrightarrow 2NO + 3Cu\ (NO_3)_2 + 4H_2O$

Dilute

$Cu + 4HNO_3 \longrightarrow 2NO_2 + Cu\ (NO_3)_2 + 2H_2O$

Conc.

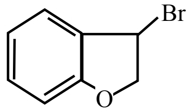
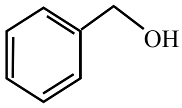
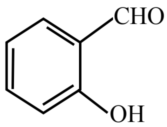
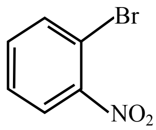
$4Zn + 10\ HNO_3 \longrightarrow N_2O + 4Zn(NO_3)_2 + 5H_2O$

Dilute

$Zn + 4HNO_3 \longrightarrow 2NO_2 + Zn\ (NO_3)_2 + 2H_2O$

Conc.

11. Match each of reactions given in **Column I** with the corresponding product(s) given in **ColumnII**.

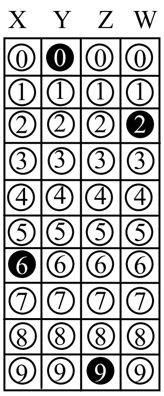
Column I	**Column II**
(A) 	(p) Nucleopilic substitution
(B) 	(q) Elimination
(C) 	(r) Nucleophilic addition
(D) 	(s) Esterification with acetic anhydride
	(t) Dehydrogenation

[Ans. (A) → p, q; (B) → p, s, t; (C) → r,s; (D) → p]

SECTION – IV

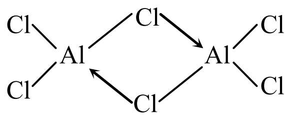
Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :



12. The coordination number of Al in the crystalline state of $AlCl_3$ is –

[Ans. 4]

Sol. 

13. The oxidation number of Mn in the product of alkaline oxidative fusion of MnO_2 is –

[Ans. 6]

Sol. $MnO_2 + KNO_2 \xrightarrow{\text{Fuse in NaOH}} K_2MnO_4 + NO$

14. The total number of α and β particles emitted in the nuclear reaction ${}^{238}_{92}U \rightarrow {}^{214}_{82}Pb$ is –

[Ans. 8]

Sol. 6α & 2β emission occurs.

Six alpha emission decreases mass number by 24 and atomic number by 12. 2β emission increases mass number by 2.

15. The dissociation constant of a substituted benzoic acid at 25ºC is 1.0×10^{-4}. The pH of a 0.01M solution of its sodium salt is –

[Ans. 8]

Sol.

$$pH = 7 + \frac{1}{2}[pKa + \log C]$$

$$= 7 + \frac{1}{2}[4 + \log 10^{-2}]$$

$$= 8$$

16. The number of water molecule(s) directly bonded to the metal centre in $CuSO_4 . 5H_2O$ is –

[Ans. 4]

Sol. $[Cu(H_2O)_4]SO_4 . H_2O$

17. At 400 K, the root mean square (rms) speed of a gas X (molecular weight = 40) is equal to the most probable speed of gas Y at 60 K. The molecular weight of the gas Y is -

[Ans. 4]

Sol.

$$C_{rms}x = C_m y$$

$$\sqrt{\frac{3R \times 400}{40}} = \sqrt{\frac{2R \times 60}{M}}$$

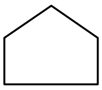
or $$30 = \frac{2 \times 60}{M}$$

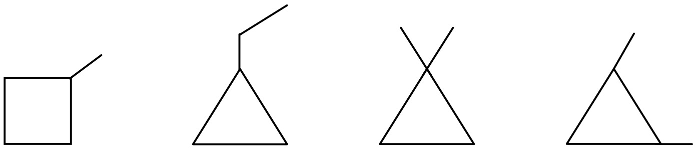
or $$M = 4$$

18. The total number of cyclic structural as well as stereo isomers possible for a compound with the molecular formula C_5H_{10} is-

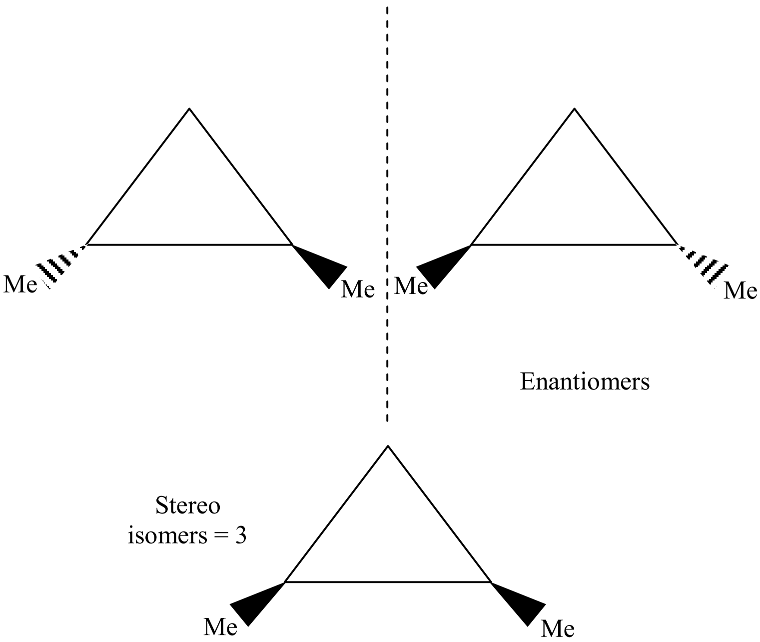
[Ans. 7]

Sol. 5 structural isomers





(3 stereo isomers are possible, these are given below)



19. In a constant volume calorimeter, 3.5 g of a gas with molecular weight 28 was burnt in excess oxygen at 298.0 K. The temperature of the calorimeter was found to increase from 298.0 K to 298.45 K due to the combustion process. Given that the heat capacity of the calorimeter is 2.5 kJ K^{-1}, the numerical value for the enthalpy of combustion of the gas in kJ mol^{-1} is-

[Ans. 9]

Sol. $Q = C\Delta T = 2.5 \times 0.45 = 1.125$ kJ

$$\Delta U = \frac{1.125 \times 28}{3.5} = 9 \text{ kJ}$$

$\Delta H = \Delta U + nR\Delta T = 9 + 1 \times 8.314 \times 0.45 \times 10^{-3} \approx 9$kJ.

Part – II (MATHEMATICS)
SECTION – I
Single Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), for its answer, out of which **ONLY ONE** is correct.

20. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x- axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points :

(A) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$

(C) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

Sol. **[C]**

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

$P \equiv (4\cos\theta, 2\sin\theta)$

$\frac{2x}{16} + \frac{2y}{4}y' = 0$ $\qquad y' = -\frac{x}{16}\cdot\frac{4}{y}$

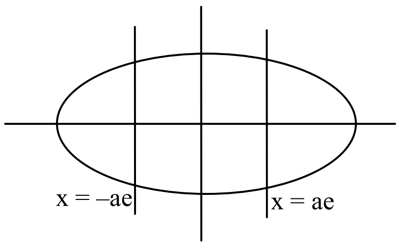
$M_N = \frac{4y}{x} = \frac{4(2\sin\theta)}{4\cos\theta} = 2\tan\theta$ $\qquad$ (M_N = slope of normal)

$y - 2\sin\theta = 2\tan\theta\,(x - 4\cos\theta)$

$Q \equiv (4\cos\theta - \cos\theta, 0) \Rightarrow Q \equiv (3\cos\theta, 0)$

$M \equiv (h, k) \equiv (7/2\cos\theta, \sin\theta)$

$\frac{4x^2}{49} + y^2 = 1$



$\because b^2 = a^2(1 - e^2) \Rightarrow e = \frac{\sqrt{3}}{2}$

$x = 2\sqrt{3}$, $x = -2\sqrt{3}$ $\qquad$ (equation of latus rectum)

$\frac{4.12}{49} + y^2 = 1 \quad y^2 = \frac{1}{49}$ $\qquad \Rightarrow y = \pm\frac{1}{7}$

21. A line with positive direction cosines passes through the point P(2, –1, 2) and makes equal angles with the coordinate axes. The line meets the plane $2x + y + z = 9$ at point Q. The length of the line segment PQ equals :

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

Sol. **[C]**

$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-2}{1} = \lambda$ $P \equiv (2, -1, 2)$

$Q \equiv (2 + \lambda, -1 + \lambda, 2 + \lambda)$

$2(2 + \lambda) + (\lambda - 1) + (\lambda + 2) = 9$

$\Rightarrow 4 + 2\lambda + \lambda - 1 + \lambda + 2 = 9$

$\Rightarrow 4\lambda = 4 \Rightarrow \lambda = 1$

$Q \equiv (3, 0, 3)$

$PQ \equiv \sqrt{1+1+1} = \sqrt{3}$

22. The locus of the orthocentre of the triangle formed by the lines

$(1 + p)x - py + p(1 + p) = 0$

$(1 + q)x - qy + q(1 + q) = 0,$

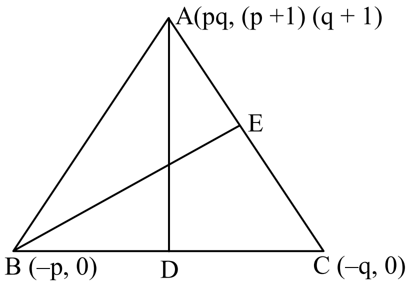
and $y = 0$, where $p \neq q$, is :

(A) a hyperbola (B) a parabola

(C) an ellipse (D) a straight line

Sol. **[D]**

Intersection points of given lines are (–p, 0), (–q, 0), [pq, (p +1) (q + 1)] respectively.



Now equation of altitudes AD and BE are $x = pq$, and $qx + (q + 1)y + pq = 0$

their point of intersection is (pq, –pq)

so, $h = pq$, $k = -pq$

so locus is $h = -k$

$h + k = 0$

$\Rightarrow x + y = 0$ which is a straight line.

23. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is :

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

Sol. [C]

$T_n = S_n - S_{n-1}$

$= cn^2 - c(n-1)^2$

$= cn^2 - cn^2 + 2cn - c$

$= 2cn - c$

$T_n^2 = c^2(2n-1)^2 = c^2(4n^2 - 4n + 1)$

$\sum T_n^2 = c^2 [4\sum n^2 - 4 \sum n + n]$

$= c^2\left[\frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n\right]$

$= \frac{nc^2}{6}[8n^2 + 12n + 4 - 12n - 12 + 6]$

$= \frac{nc^2}{6}[8n^2 - 2]$

$= \frac{nc^2(4n^2-1)}{3}$

SECTION – II

Multiple Correct Choice Type

This section contains 5 Multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), for its answer, out of which ONE OR MORE is/are correct.

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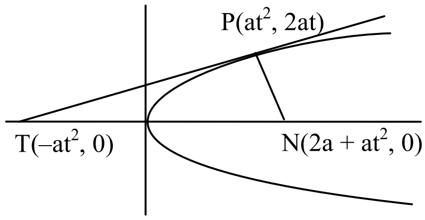
24. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose :

(A) vertex is $\left(\frac{2a}{3}, 0\right)$ (B) directrix is $x = 0$

(C) latus rectum is $\frac{2a}{3}$ (D) focus is $(a, 0)$

Sol. **[A, D]**



$ty = x + at^2$

$y = -tx + 2at + at^3$

$h = \frac{at^2 - at^2 + 2a + at^2}{3}$, $k = \frac{2at + 0 + 0}{3}$

$3h = a\,(2 + t^2)$, $\qquad t = \frac{3k}{2a}$

$\Rightarrow 3h = a\left(2 + \frac{9k^2}{4a^2}\right)$

$\Rightarrow 12\,ah = 8a^2 + 9k^2$

$\Rightarrow 9y^2 = 12ax - 8a^2$

$\Rightarrow y^2 = \frac{4a}{3}\left(x - \frac{2}{3}a\right)$

vertex $\left(\frac{2a}{3}, 0\right)$ directrix $x = \frac{a}{3}$

Latus rectum $\frac{4a}{3}$, Focus (a, 0)

25. For $0 < \theta < \frac{\pi}{2}$, the solution (s) of

$$\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$$

is (are) :

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

Sol. **[C, D]**

$$\frac{1}{\sin(\pi/4)}\sum_{m=1}^{6}\frac{\sin\left[\left(\theta+\frac{m\pi}{4}\right)-\left(\theta+\frac{(m-1)\pi}{4}\right)\right]}{\sin\left(\theta+\frac{(m-1)\pi}{4}\right)\sin\left(\theta+\frac{m\pi}{4}\right)}$$

$$\frac{1}{\sin(\pi/4)}\sum_{m=1}^{6}\left[\cot\left(\theta+\frac{(m-1)\pi}{4}\right)-\cot\left(\theta+\frac{m\pi}{4}\right)\right]$$

$$\Rightarrow \frac{1}{\sin\pi/4}\left[\cot\theta-\cot\left(\theta+\frac{\pi}{4}\right)\right]+\left[\cot\left(\theta+\frac{\pi}{4}\right)-\cot\left(\theta+\frac{2\pi}{4}\right)\right]+\ldots\ldots\ldots+$$

$$\left[\cot\left(\theta+\frac{5\pi}{4}\right)-\cot\left(\theta+\frac{6\pi}{4}\right)\right]$$

$$=\frac{1}{\sin\pi/4}\left[\cot\theta-\cot\left(\theta+\frac{3\pi}{2}\right)\right]=4\sqrt{2}$$

$\cot\theta+\tan\theta=4$

$$\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=4$$

$$\sin 2\theta=\frac{1}{2}$$

$$\theta=\frac{\pi}{12},\ \frac{5\pi}{12}$$

26. For the function $f(x) = x\cos\frac{1}{x}$, $x \ge 1$,

(A) for at least one x in the interval $[1, \infty)$, $f(x + 2) - f(x) < 2$

(B) $\lim_{x\to\infty} f'(x) = 1$

(C) for all x in the interval $[1, \infty)$, $f(x + 2) - f(x) > 2$

(D) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

Sol. **[B,C, D]**

$$f(x) = x\cos\left(\frac{1}{x}\right)$$

$$f'(x) = \cos\left(\frac{1}{x}\right) + \frac{\sin(1/x)}{x}$$

(B)

$$\lim_{x\to\infty} f'(x) = \lim_{x\to\infty}\cos(1/x) + \lim_{x\to\infty}\frac{\sin(1/x)}{x}$$

$= 1 + 0 = 1$

(D) $f''(x) = \frac{-\cos(1/x)}{x^3}$

$\because$ for $x \in [1, \infty)$ $f''(x)$ is negative.

so $f'(x)$ is decreasing for $x \in [1, \infty)$

using LMVT

$$\frac{f(x+2)-f(x)}{x+2-x} = f'(x)$$

$$= \cos\frac{1}{x} + \frac{1}{x}\sin\left(\frac{1}{x}\right)$$

$\because$ $f''(x) < 0$ so $f'(x)$ is decreasing

$$f'(x) > \lim_{x\to\infty} f'(x)$$

$$f'(x) > 1$$

so $\frac{f(x+2)-f(x)}{x+2-x} > 1$

$$f(x+2) - f(x) > 2$$

$$f(x+2) - f(x) - 2 > 0$$

27. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then :

(A) Equation of ellipse is $x^2 + 2y^2 = 2$

(B) The foci of ellipse are $(\pm 1, 0)$

(C) Equation of ellipse is $x^2 + 2y^2 = 4$

(D) The foci of ellipse are $(\pm\sqrt{2}, 0)$

Sol. **[A, B]**

Since both conic cuts orthogonally so foci coincides

Now foci of hyperbola is $(\pm 1, 0)$

$\therefore$ foci of ellipse $(\pm 1, 0)$

$\therefore$ eccentricity of rectangular hyperbola is $\sqrt{2}$

$\therefore$ eccentricity of ellipse $= \frac{1}{\sqrt{2}}$

$\therefore$ equation of ellipse is $\frac{x^2}{2} + y^2 = 1$

28. If

$$I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, \; n = 0, 1, 2, \ldots\ldots,$$

then :

(A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

(C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

Sol. $I_n = \int_{-\pi}^{\pi} \frac{\sin n\ x}{(1+\pi^x)\sin x}$

$\Rightarrow I_n = \int_{-\pi}^{\pi} \frac{\pi^x \sin nx}{(1+\pi^x)\sin x} dx$

Adding $2I_n = \int_{-\pi}^{\pi} \frac{\sin(nx)}{\sin x} dx$

or $I_n = \int_{0}^{\pi} \frac{\sin nx}{\sin x} dx$

$\Rightarrow I_{n+2} = \int_{0}^{\pi} \frac{\sin(n+2)x}{\sin x} dx$

Consider

$I_{n+2} - I_n = \int_{0}^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x} dx$

$= \int_{0}^{\pi} \frac{2\cos(n+1)x\ \sin x}{\sin x} dx = 0$

$\Rightarrow I_{n+2} = I_n$ … (1)

again $\sum_{m=1}^{10} I_{2m+1} = I_3 + I_5 + I_7 + \ldots.. I_{21}$

using (1) $= 10I_3 = 10I_1$

$= 10\pi$

$\sum_{m=1}^{10} I_{2m} = 0$ because

$I_{2m} = \int_{0}^{\pi} \frac{\sin 2mx}{\sin x} dx = 0$ use $f(a - x) = -f(x)$

Also $I_{n+1} \neq I_n$

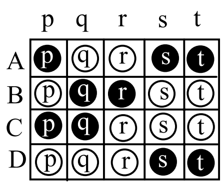
as by trial $I_1 \neq I_2$

SECTION – III

Matrix - Match Type

This section contains 2 questions. Each question contains statements given in two columns which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p,q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement (s) in **Column II.** The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A -p , s and t; B -q and r; C -p and q; and D –s and t; then the correct darkening of bubbles will look line the following .



29. Match the statements/expressions given in **Column I** with the values given in **Column II.**

Column I	Column II
(A) Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(p) $\frac{\pi}{6}$
(B) Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, where[y] denotes the largest integer less than or equal to y	(q) $\frac{\pi}{4}$
(C) Volume of the parallelepiped with its edges represented by the vectors $\hat{i} + \hat{j}$, $\hat{i} + 2\hat{j}$ and $\hat{i} + \hat{j} + \pi\hat{k}$	(r) $\frac{\pi}{3}$
(D) Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a} + \vec{b} + \sqrt{3}\,\vec{c} = \vec{0}$	(s) $\frac{\pi}{2}$
	(t) π

Sol. A $\rightarrow$ q, s; B $\rightarrow$ p, r, s, t; C $\rightarrow$ t; D $\rightarrow$ r

(A)

$2\sin^2\theta + \sin^2 2\theta = 2$

$\Rightarrow 2\sin^2\theta + 4\sin^2\theta\cos^2\theta - 2 = 0$

$\Rightarrow \sin^2\theta + 2\sin^2\theta(1 - \sin^2\theta) - 1 = 0$

$\Rightarrow 2\sin^4\theta - 3\sin^2\theta + 1 = 0$

$\Rightarrow (2\sin^2\theta - 1)(\sin^2\theta - 1) = 0$

$\Rightarrow \sin^2\theta = \frac{1}{2}$ or $\sin^2\theta = 1$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{2}$

(B)

$f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$

at $x = \frac{\pi}{6} \Rightarrow \left[\frac{6x}{\pi}\right] = [1] = 1$ & $\left[\frac{3x}{\pi}\right] = \left[\frac{1}{2}\right] = 0$

at $x = \frac{\pi}{3} \Rightarrow \left[\frac{6x}{\pi}\right] = [2] = 2$ & $\left[\frac{3x}{\pi}\right] = [1] = 1$

So point of discontinuity

$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$

(C)

Volume of parallelepiped = $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = \pi$

(D)

$\sqrt{3}\,\vec{c} = -\vec{a} - \vec{b}$

$3|\vec{c}|^2 = (-\vec{a} - \vec{b}).(-\vec{a} - \vec{b})$

$3 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}|.|\vec{b}|\cos\theta$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

30. Match the statements/expressions given in **Column I** with the values given in **Column II.**

Column I	**Column II**
(A) The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p) 1
(B) Value (s) of k for which the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	(q) 2
(C) Value(s) of k for which $\lvert x - 1\rvert + \lvert x - 2\rvert + \lvert x + 1\rvert + \lvert x + 2\rvert = 4k$ has integer solution (s)	(r) 3
(D) If $y' = y + 1$ and $y(0) = 1$, then value (s) of $y(\ln 2)$	(s) 4
	(t) 5

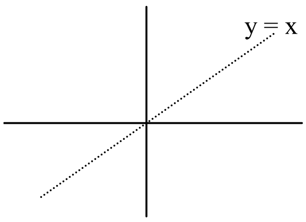
Sol. $A \to p$; $B \to q, s$; $C \to q, r, s, t$; $D \to r$

(A)

$f(x) = xe^{\sin x} - \cos x$

$f'(x) = e^{\sin x} + xe^{\sin x} \cos x + \sin x > 0$ in $(0, \pi/2)$

$\Rightarrow f(x)$ increasing in $x \in (0, \pi/2)$



$f(0) = -1 < 0$

$f(\pi/2) = \frac{\pi}{2}e - 1 > 0$

$\Rightarrow$ 1 solution

(B)

$$\begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$k(k - 4) - 4(4 - 4) + (8 - 2k) = 0$

$k(k - 4) - 2(k - 4) = 0 \Rightarrow (k - 2)(k - 4) = 0 \Rightarrow k = 2, 4$

(C)

If $x \geq 2 \Rightarrow 4x = 4k \Rightarrow x = k \quad (k = 2, 3, 4, 5)$

If $1 \leq x < 2 \quad \Rightarrow x - 1 + 2 - x + 2x + 3 = 4k$

$2x + 4 = 4k;$

$x = 1 \Rightarrow 4k = 6$ (Not possible)

$-1 \leq x < 1 \quad \Rightarrow 1 - x + 2 - x + x + 1 + x + 2 = 4k$

$\Rightarrow 6 = 4k$ (Not possible)

$-2 \leq x < -1 \quad \Rightarrow \quad 1 - x + 2 - x - 1 - x + x + 2 = 4k$

$-x + 4 = 4k$ (Not possible)

$x < -2 \Rightarrow 1 - x + 2 - x - 1 - x - x - 2 = 4k$

$-4x = 4k \Rightarrow x = -k \quad \Rightarrow k = 3$

(D)

$\frac{dy}{dx} = y + 1 \Rightarrow \int \frac{dy}{y+1} = \int dx + c$

$\ell n\,(y + 1) = x + c$

$y(0) = 1 \qquad \Rightarrow \ell n\, 2 = c$

$\ell n\,(y + 1) = x + \ell n\, 2$

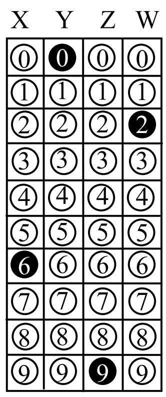
$y + 1 = 2e^{x}$

$y\,(\ell n\, 2) = 2e^{\ell n 2} - 1 = 3$

SECTION – IV

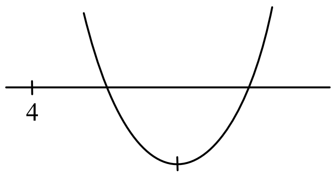
Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :



31. The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is :

Sol. [2]



$D > 0 \Rightarrow 64k^2 - 64(k^2 - k + 1) > 0$

$64k^2 - 64k^2 + 64k - 64 > 0$

$k > 1$

$-\frac{B}{2A} > 4 \Rightarrow 4k > 4 \Rightarrow k > 1$

$f(4) \geq 0$

$16 - 32k + 16k^2 - 16k + 16 \geq 0 \Rightarrow 16k^2 - 48k + 32 \geq 0$

$k^2 - 3k + 2 \geq 0 \Rightarrow (k-1)(k-2) \geq 0 \Rightarrow k \leq 1, k \geq 2$

so $k \in [2, \infty)$

so smallest integer value of k is 2.

32. Let $f: R \rightarrow R$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then the value of $f(\ln 5)$ is :

Sol. [0]

$f'(x) = f(x) \Rightarrow \frac{f'(x)}{f(x)} = 1$

$\int \frac{f'(x)}{f(x)} dx = \int dx \Rightarrow \ell n\,[f(x)] = x + c$

$f(x) = e^{x+c} \Rightarrow f(x) = Ae^x \Rightarrow$ as $f(0) = 0$ (Given)

So $A = 0 \quad \therefore f(x) = 0 \Rightarrow f(\ell n\, 5) = 0$

33. Let p(x) be a polynomial of degree 4 having extremum at x = 1,2 and $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right) = 2$. Then the value of p(2) is :

Sol. [0]

Let $P(x) = ax^4 + bx^3 + cx^2 + dx + e$

$\because \lim_{x\to 0}\left(1+\frac{P(x)}{x^2}\right) = 2$

so $\lim_{x\to 0}\frac{P(x)}{x^2} = 1 \Rightarrow \lim_{x\to 0} ax^2 + bx + c + \frac{d}{x} + \frac{e}{x^2} = 1$

$\Rightarrow d = e = 0 \ \&\ c = 1$

so $P(x) = ax^4 + bx^3 + x^2$

$\because P'(x) = 4ax^3 + 3bx^2 + 2x$

$\because P'(1) = P'(2) = 0$ so

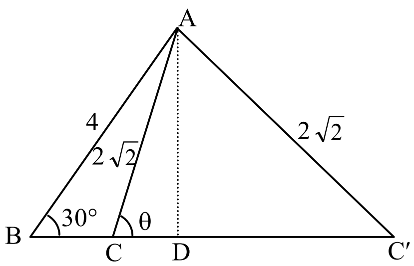
$4a + 3b + 2 = 0 \ \&\ 32a + 12b + 4 = 0$

On solving $a = \frac{+1}{4}$ & $b = -1$

$P(2) = 16a + 8b + 4c = 0$

34. Let ABC and ABC' be two non-congruent triangles with sides AB = 4, AC = AC' = $2\sqrt{2}$ and angle B = 30º. The absolute value of the difference between the areas of these triangles is :

Sol. [4]



$AD = 4 \sin 30° = 2$

difference of areas = $\Delta ACC'$

$2 = 2\sqrt{2} \sin\theta \Rightarrow \theta = 45°$

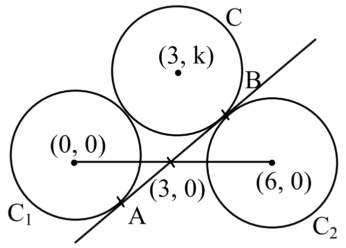
side $CC' = 2\,(2\sqrt{2}\cos 45°) = 4$

$\Delta ACC' = \frac{1}{2} \cdot 4 \cdot 2 = 4$

35. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is :

Sol. [8]

Let the coordinate system is as follows



equation of AB is $y = m(x - 3)$

$\because$ AB is tangent to C_1 so $m = \pm\dfrac{1}{2\sqrt{2}}$

But m should be positive $m = \dfrac{1}{2\sqrt{2}}$

So equation of AB = $2\sqrt{2} = x - 3$

$\because$ C_1 & C are touching each other externally.

So $CC_1 = r_1 + r \Rightarrow 9 + k^2 = (r + 1)^2$ … (1)

$\because$ AB is tangent to circle C so

$r = \dfrac{|2\sqrt{2}k - 3 + 3|}{\sqrt{8+1}} \Rightarrow k = \dfrac{3r}{2\sqrt{2}}$ … (2)

So solving (1) and (2) $r = 8$

36. Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations :

$$3x - y - z = 0, \quad -3x + z = 0, \quad -3x + 2y + z = 0.$$

Then the number of such points for which $x^2 + y^2 + z^2 \leq 100$ is :

Sol. [7]

$\because$ $3x - y - z = 0$, $3x - z = 0$, $3x - 2y - z = 0$

On solving these three $y = 0$

$z = 3x$

so $x^2 + y^2 + z^2 \leq 100$

$x^2 + 0 + 9x^2 \leq 100$

$x^2 \leq 10 \Rightarrow |x| = 0, 1, 2, 3$

so total no. of different points possible are 7

(0, 0, 0), (–1, 0, 1), (–1, 0, –1), (2, 0, 2), (–2, 0, –2), (3, 0, 3), (–3, 0, –3)

37. If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is :

Sol. [2]

$g(x) = f^{-1}(x)$

$f(g(x)) = x$

$\Rightarrow g(f(x)) = x \Rightarrow g'(f(x))\, f'(x) = 1 \quad \Rightarrow g'(f(x)) = \frac{1}{f'(x)}$

put x = 0

$\Rightarrow g'(1) = \frac{1}{f'(0)} \Rightarrow \quad f'(x) = 3x^2 + \frac{1}{2}e^{x/2}$

$\therefore f'(0) = \frac{1}{2} \quad \Rightarrow \therefore g'(1) = 2$

38. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x | x^2 + 20 \le 9x\}$ is :

Sol. [7]

$f(x) = 2x^3 - 15x^2 + 36x - 48$

$f'(x) = 6(x - 2)(x - 3)$

$\because A = \{x | x^2 + 20 - 9x \le 0\}$

$4 \le x \le 5$

so f(x) is increasing for $x \in [3, \infty)$

so $(f(x))_{max}$ at $x \in [4, 5]$ is f(5)

so $(f(x))_{max} = f(5) = 7$

Part – III (PHYSICS)

SECTION – I

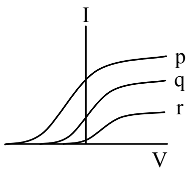
Single Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

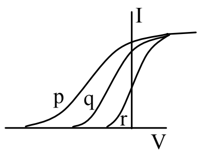
39. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions ϕ_p = 2.0 eV, ϕ_q = 2.5 eV and ϕ_r = 3.0 eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is :

[Take hc = 1240 eV nm]

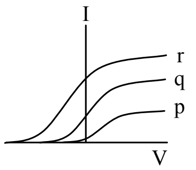
(A)



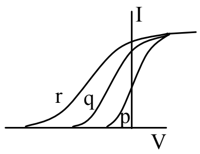
(B)



(C)



(D)



Ans.[A]

Sol. $E_{550\text{ nm}} = \dfrac{12400}{5500} = 2.25$ eV

$E_{450\text{ nm}} = \dfrac{12400}{4500} = 2.75$ eV

$E_{350} = \dfrac{12400}{3500} = 3.54$ eV

$eV_s = E - \phi$

For metal p, $V_s = 3.54 - 2$

$V_s = 1.54$ V

For metal q, $V_s = 3.54 - 2.5$

$= 1.04$ V

For metal r, $V_s = 3.54 - 3$

$= 0.54$ V

hence option 'A' is correct.

Also $I_p > I_q > I_r$

40. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is –

(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$ (C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

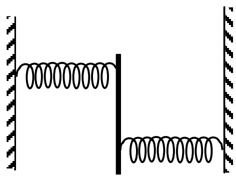
Ans.[B]

Sol. $a = g\tan\theta$ where $\tan\theta = \frac{dy}{dx} = 2kx$

$\therefore a = 2\,kx\,.\,g$

$\therefore x = \frac{a}{2kg}$

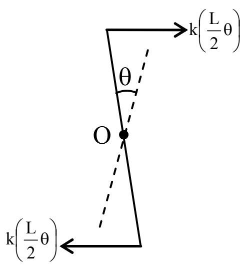
41. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is –



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

Ans.[C]

Sol.



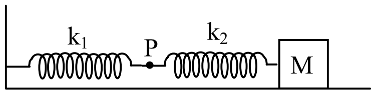
Take τ torque about 'O'

$2.\ k\frac{L}{2}\theta.\frac{L}{2} = -\frac{ML^2}{12}.\alpha$

$\therefore \alpha = -\frac{6k}{M}.\theta$

$\therefore f = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

42. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is –



(A) $\frac{k_1A}{k_2}$ (B) $\frac{k_2A}{k_1}$ (C) $\frac{k_1A}{k_1+k_2}$ (D) $\frac{k_2A}{k_1+k_2}$

Ans.[D]

Sol. $x_1 + x_2 = A$ and $k_1x_1 = k_2x_2$

$\therefore\ x_1 + \frac{k_1x_1}{k_2} = A$

$\therefore\ x_1 = \frac{k_2A}{(k_1+k_2)}$

SECTION – II

Multiple Correct Choice Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

43. Under the influence of the Coulomb field of charge +Q, a charge –q is moving around it in an elliptical orbital. Find out the correct statement(s).

(A) The angular momentum of the charge – q is constant

(B) The linear momentum of the charge –q is constant

(C) The angular velocity of the charge –q is constant

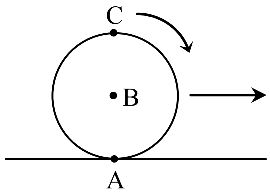
(D) The linear speed of the charge –q is constant

Ans.[A]

Sol. Torque of coulombic force on '–q' about 'Q' is zero.

* Angular momentum of without specifying a point about which it is to be calculated, do not have significance.

44. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then –



(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$

(B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

(D) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

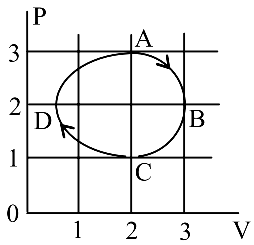
Ans.[B, C]

Sol. $\vec{V}_A = 0$

$\vec{V}_B = \vec{V}$ (Let)

$\vec{V}_C = 2\vec{V}$

45. The figure shows the P–V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semi-circle and CDA is half of an ellipse. Then –



(A) the process during the path A → B is isothermal

(B) heat flows out of the gas during the path B → C → D

(C) work done during the path A → B → C is zero

(D) positive work is done by the gas in the cycle ABCDA

Ans.[B, D]

Sol. **(B)** ΔQ = ΔU + W

In BCD : W is negative

$\Delta U = \frac{P_f V_f - P_i V_i}{(\gamma - 1)} = -ve$

(D) Cycle is clockwise.

46. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air-column is the second resonance. Then -

(A) the intensity of the sound heard at the first resonance was more than that at the second resonance

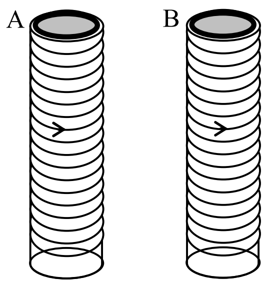
(B) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube

(C) the amplitude of vibration of the ends of the prongs is typically around 1 cm

(D) the length of the air-column at the first resonance was somewhat shorter than $1/4^{th}$ of the wavelength of the sound in air

Ans.[A,D]

47. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is (are) –



(A) $\rho_A > \rho_B$ and $m_A = m_B$

(B) $\rho_A < \rho_B$ and $m_A = m_B$

(C) $\rho_A > \rho_B$ and $m_A > m_B$

(D) $\rho_A < \rho_B$ and $m_A < m_B$

Ans.[B,D]

Sol. $B = \mu_0 ni$

$\phi = \mu_0 ni \times A$

where A = Area of ring

$e = \frac{\mu_0 ni \times A}{\Delta t}$

induced current = I = $\frac{\mu_0 n i A \times (\text{area})}{\Delta t \rho_A \times L}$

Force due to induced current in ring is

For short duration is Δt

Impulse = F Δt

Impulse $\propto$ induced current

$m\Delta v \propto \frac{1}{\rho}$

$v \propto \frac{1}{\rho m}$

so maximum height depend on v

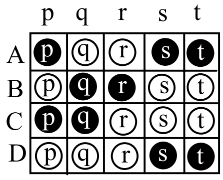
$\therefore h_{max} \propto \frac{1}{\sqrt{\rho m}}$

SECTION – III

Matrix-Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement **in Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example.

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following.

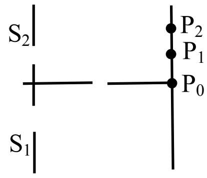
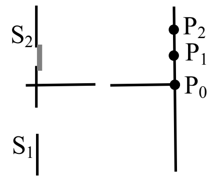
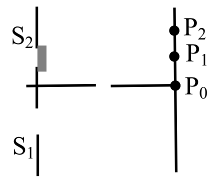
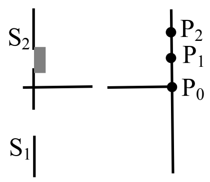


48. **Column II** gives certain systems undergoing a process. **Column I** suggests changes in some of the parameters related to the system. Match the statements in **Column I** to the appropriate process(es) from **Column II**.

Column I	**Column II**	
(A) The energy of the system is increased	(p) System :	A capacitor, initially uncharged
	Process :	It is connected to a battery
(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts	(q) System :	A gas in an adiabatic container fitted with an adiabatic piston
	Process :	The gas is compressed by pushing the piston
(C) Internal energy of the system is converted into its mechanical energy	(r) System :	A gas in a rigid container
	Process :	The gas gets cooled due to colder atmosphere surrounding it
(D) Mass of the system is decreased	(s) System:	A heavy nucleus,initially at rest
	Process :	The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	(t) System :	A resistive wire loop
	Process :	The loop is placed in a time varying magnetic field perpendicular to its plane

Sol. **A $\rightarrow$ p,q,s,t ; B $\rightarrow$ q ; C $\rightarrow$ s, D $\rightarrow$ s**

49. **Column I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in **Column I** with the statement(s) in **Column II** valid for that situation.

Column I		**Column II**
(A)		(p) $\delta(P_0) = 0$
(B) $(\mu - 1)t = \lambda/4$		(q) $\delta(P_1) = 0$
(C) $(\mu - 1)t = \lambda/2$		(r) $I(P_1) = 0$
(D) $(\mu - 1)t = 3\lambda/4$		(s) $I(P_0) > I(P_1)$
		(t) $I(P_2) > I(P_1)$

Sol. **A → p,s ; B → q ; C → t ; D → r, s t**

(A) At P_0

$\delta(P_0) = 0$

$I(P_0) = 4I_0$

At P_1

$\delta(P_1) = \frac{\lambda}{4}$

$I(P_1) = 2I_0$

$I(P_0) > I(P_1)$

At P_2

$\delta(P_2) = \frac{\lambda}{3}$

$\phi = \frac{2\pi}{\lambda}\times\frac{\lambda}{3} = \frac{2\pi}{3}$

$I(P_2) = 4I_0\cos^2\frac{2\pi}{6} = I_0$

$I(P_2) > I(P_1)$

(B) $\delta(P_0) = \frac{\lambda}{4}$

$I(P_0) = 2I_0$

$\delta(P_1) = 0$

$I(P_1) = 4I_0$

$\delta(P_2) = \frac{\lambda}{3}-\frac{\lambda}{4} = \frac{\lambda}{12}$

$\phi = \frac{2\pi}{\lambda}\times\frac{\lambda}{12} = \frac{\pi}{6}$

$I(P_1) = 2I_0(1 + \cos\frac{\pi}{6})$

$= 2I_0(1 + \frac{\sqrt{3}}{2})$

$= 2I_0 + \sqrt{3}\,I_0$

$I(P_1) > I(P_2)$

(C) $\delta(P_0) = \frac{\lambda}{2}$

$I(P_0) = 0$

$\delta(P_1) = \frac{\lambda}{2} - \frac{\lambda}{4} = \frac{\lambda}{4}$

$I(P_1) = 2I_0$

$\delta(P_2) = \frac{\lambda}{2} - \frac{\lambda}{3} = \frac{\lambda}{6}$

$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$

$I(P_2) = 4I_0 \cos^2 \frac{\pi}{6}$

$= 3I_0$

(D) $\delta(P_0) = \frac{3\lambda}{4}$

$I(P_0) = 2I_0$

$\delta(P_1) = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$

$I(P_1) = 0$

$\delta(P_2) = \frac{3\lambda}{4} - \frac{\lambda}{3} = \frac{9\lambda - 4\lambda}{12} = \frac{5\lambda}{12}$

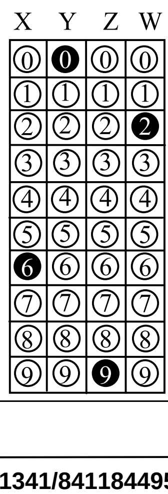
$I(P_2) \neq 0$

$I(P_2) > I(P_1)$

SECTION – IV

Integer Answer Type

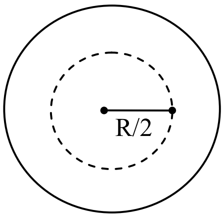
This section contains 8 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :



50. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = \kappa r^a$, where κ and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at r = R, find the value of a.

Ans. [2]

Sol.



for $r \le \frac{R}{2}$

$$Q_{in} = \int_0^{R/2} \rho 4\pi r^2 \, dr$$

$$= 4\pi \int_0^{R/2} k\, r^{a+2} \, dr$$

$$= 4\pi k \int_0^{R/2} r^{a+2} \, dr$$

$$= \frac{4\pi k}{(a+3)} \left[r^{a+3} \right]_0^{R/2}$$

$$Q_{in} = \frac{4\pi k}{(a+3)} \left(\frac{R}{2} \right)^{a+3}$$

$$E\left(r = \frac{R}{2} \right) = \frac{\frac{4\pi k}{(a+3)} \left(\frac{R}{2} \right)^{a+3}}{4\pi \left(\frac{R}{2} \right)^2}$$

For $r \le R$

$$Q_{in} = \frac{4\pi k}{(a+3)} R^{a+3}$$

$$E(r = R) = \frac{\frac{4\pi k}{(a+3)} R^{a+3}}{4\pi (R)^2}$$

$$E\left(r=\frac{R}{2}\right) = \frac{1}{8}\ E(r=R)$$

$$\Rightarrow \frac{\frac{4\pi k}{(a+3)}\left(\frac{R}{2}\right)^{a+3}}{4\pi\left(\frac{R}{2}\right)^2} = \frac{1}{8}\,\frac{\frac{4\pi k}{(a+3)}R^{a+3}}{4\pi(R)^2}$$

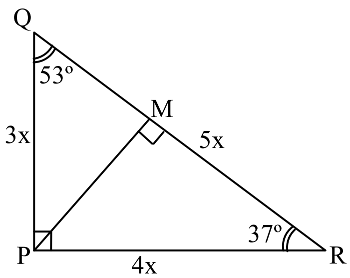
$$\left(\frac{1}{2}\right)^{a+1} = \left(\frac{1}{2}\right)^3$$

$a = 2$

51. A steady current I goes through a wire loop PQR having shape of a right angle triangle with PQ = 3x, PR = 4x and QR = 5x. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k.

Ans. [7]

Sol.



$$\sin 37^\circ = \frac{PM}{4x}$$

$$PM = \frac{3}{5} \times 4x = \frac{12x}{5}$$

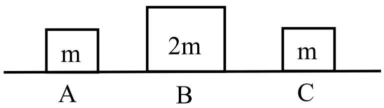
$$B_P = \frac{\mu_0 I}{4\pi\left(\frac{12x}{5}\right)}\ [\sin 53^\circ + \sin 37^\circ]$$

$$= \frac{\mu_0 I \times 5}{4\pi(12x)}\left[\frac{4}{5}+\frac{3}{5}\right]$$

$$= \frac{\mu_0 I \times 5}{48\pi x}\,\frac{7}{5} = 7\left(\frac{\mu_0 I}{48\pi x}\right)$$

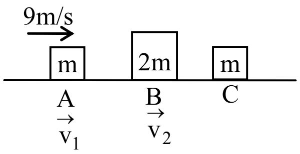
$k = 7$

52. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C.



Ans. [4]

Sol.



Collision between A and B,

$m \times 9 = mv_1 + 2mv_2$

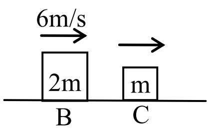
$9 = v_1 + 2v_2$ ....(1)

$e = 1 = \frac{(v_2 - v_1)}{9}$

$v_2 - v_1 = 9$ ....(2)

from (1) and (2)

$v_2 = 6$ m/s



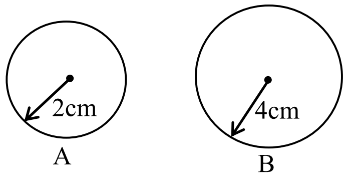
$2m \times 6 = 3m \times v_3$

$v_3 = 4$ m/s

53. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.]

Ans. [6]

Sol.



$P_A = P_{atm} + \frac{4T}{r_A} = 8 + \frac{4 \times 0.04}{0.02}$

$P_A = 16\ N/m^2$

$P_B = P_{atm} + \frac{4T}{r_B} = 8 + \frac{4 \times 0.04}{0.04}$

$P_A = 12\ N/m^2$

For ideal gas

$PV = nRT$

$P_A V_A = n_A RT$

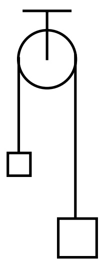
$n_A = \frac{P_A V_A}{RT}$

$n_B = \frac{P_B V_B}{RT}$

$\frac{n_B}{n_A} = \frac{P_B V_B}{P_A V_A} = \frac{12 \times \frac{4}{3}\pi(0.04)^3}{16 \times \frac{4}{3}\pi(0.02)^3}$

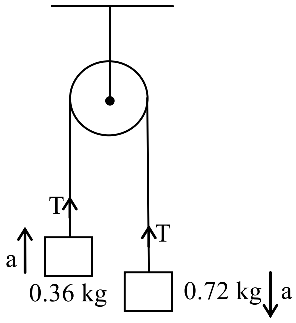
$= \frac{3}{4} \times (2)^3 = 6$

54. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10\ m/s^2$, find the work done (in joules) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.



Ans. [8]

Sol.



$a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$

$= \frac{(0.72 - 0.36)g}{(0.72 + 0.36)}$

$a = \frac{g}{3}$

$S = \frac{1}{2}at^2 = \frac{1}{2} \times \frac{g}{3}(1)^2 = \frac{g}{6}$

$T = 0.36(g + a)$

$= 0.36(g + \frac{g}{3})$

$= \frac{0.36 \times 4g}{3} = 0.12 \times 4 \times 10$

$T = 4.8$ N

Work done by tension = $4.8 \times \frac{10}{6}$

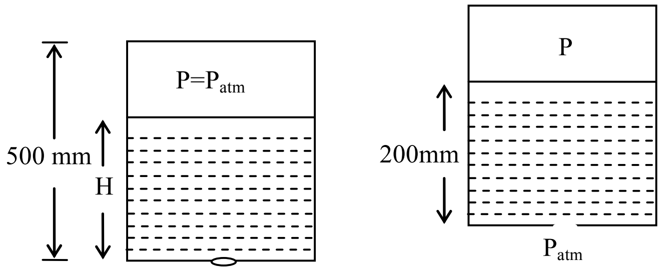
W = 8 J

55. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.

[Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.]

Ans. [6]

Sol.



Final pressure inside two cylinder,

$P_f = P_{atm} - \rho gh$

$P_f = 10^5 - 1000 \times 10 \times 0.2$

$P_f = 10^5 - 0.02 \times 10^5$

$P_f = 0.98 \times 10^5$

By Boyle's Law for gas

$P_iV_i = P_f V_f$

$10^5(500 - H) = 0.98 \times 10^5 \times 300$

$(500 - H) = 294$

$H = 500 - 294$

$H = 206$ mm

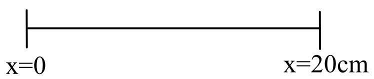
Fall in height = 206 – 200

= 6 mm

56. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string.

Ans. [5]

Sol.



T = 0.5 N

$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{0.5\times0.2}{10^{-3}}} = \sqrt{0.1\times10^3} = 10$ m/s

$v = \lambda f$

$\left[\lambda = \frac{v}{f} = \frac{10}{100} = 0.1\right]$

Separation between successive node

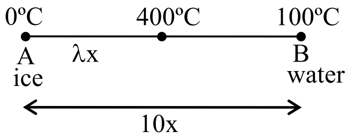
$$= \frac{\lambda}{2} = \frac{10}{2} = 5 \text{ cm}$$

57. A metal rod AB of length 10x has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ.

[Neglect any heat loss to the surrounding.]

Ans. [9]

Sol.



For ice

$$m \times 80 = \frac{k \times 400 \times A}{\lambda x} \quad ....(1)$$

For water

$$m \times 540 = \frac{k \times 300 \times A}{(10x - \lambda x)} \quad ....(2)$$

eq.(1)/eq.(2)

$$\frac{80}{540} = \frac{400}{300} \times \frac{(10-\lambda)}{\lambda}$$

$$4\lambda = \frac{4 \times 27}{3}(10 - \lambda)$$

$$\lambda = 90 - 9\lambda$$

$$10\lambda = 90 \Rightarrow \lambda = 9$$